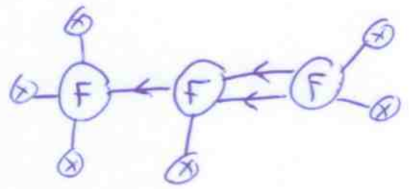
Preuve graphique de la syzygie d'Hermite : (par Malek)

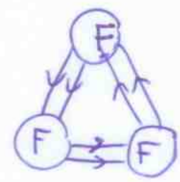
Je suis les conventions et notations de mon article dans JKTR 2012.

Forme quartique $F(x_1, x_2) =$  ou simplement F dans les calculs.

Les autres covariants sont

$H =$  la hessienne

$T =$  le cubicovariant

et les invariants $I =$  et $J =$  .

J'utiliserai une notation abregee : 

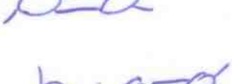
les $\otimes$ ne seront pas representes

 (deux lignes ∑ paralleles orientée dans le même sens si les flèches sont non explicits)

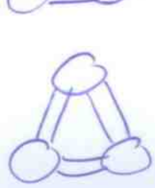
Ainsi :

$F =$

$H =$ 

$T =$ 

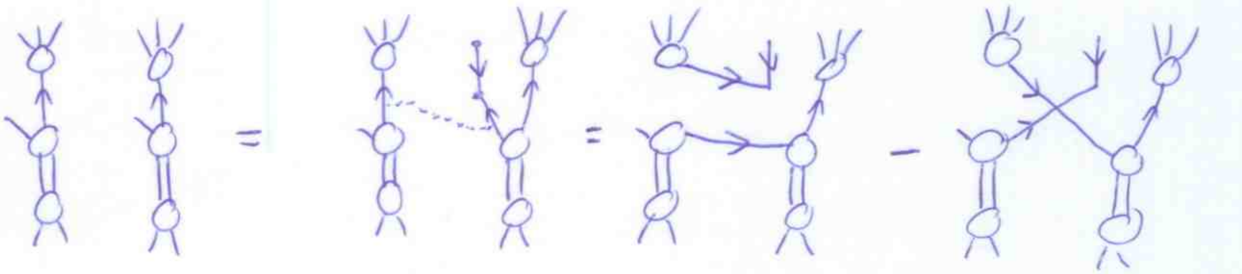
$I =$ 

$J =$ 

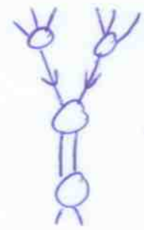
Identité de base (Grassmann-Plücker) :

(GP)

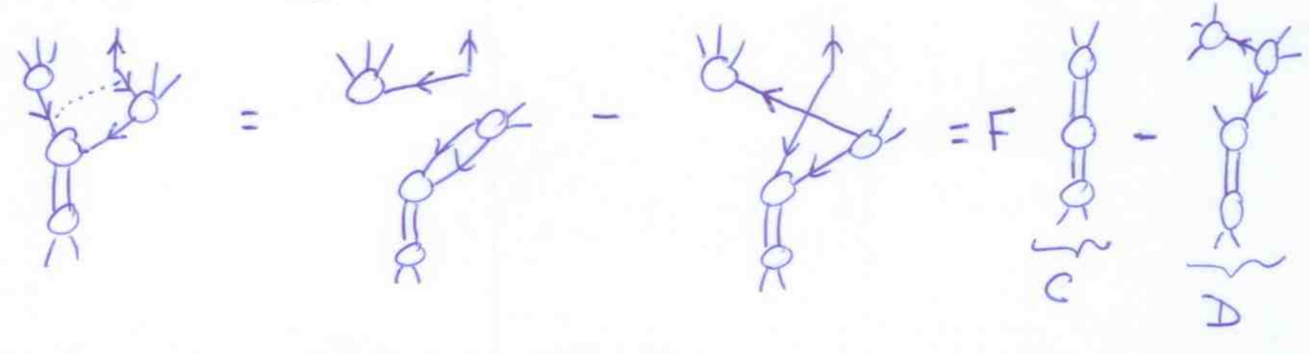
<u>Calcul</u> :

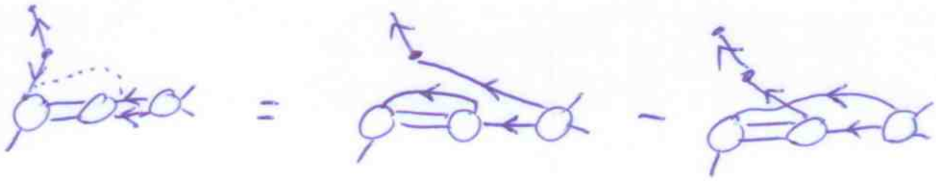
$T^2 =$ 

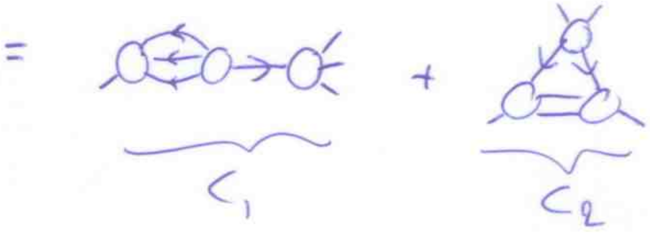
→ $Id = \varepsilon^T \varepsilon$, les pointillé indiquent la paire de lignes ε où l'identité GP sera appliqué.

$\Rightarrow T^2 = F A + H B$ avec $A =$  et $B =$  .

On travaille maintenant sur B :

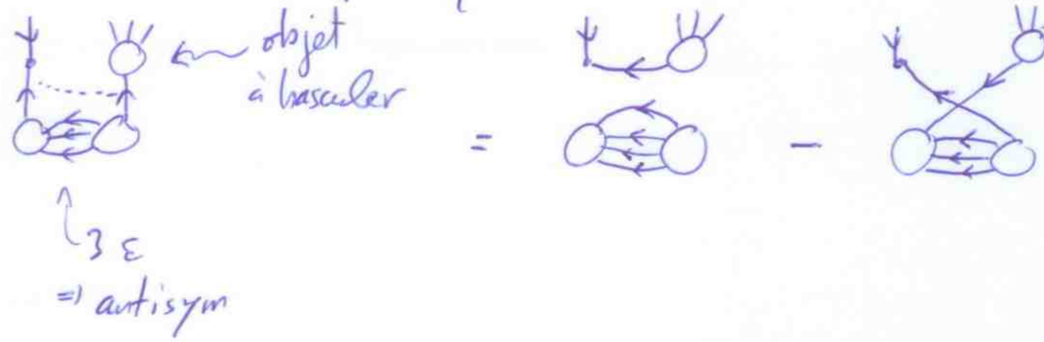
$B =$ 

$C =$ 



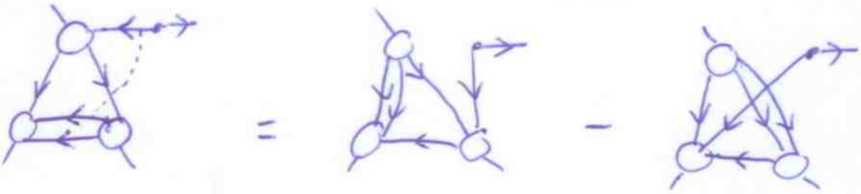
Le calcul de C_1 emploie une idée qui reviendra souvent que j'appelle la "bascule antisymetique":

$C_1 =$



$\Rightarrow \quad C_1 = FI - C_1 \quad \Rightarrow \quad \boxed{C_1 = \frac{1}{2} FI}$

$C_2 =$

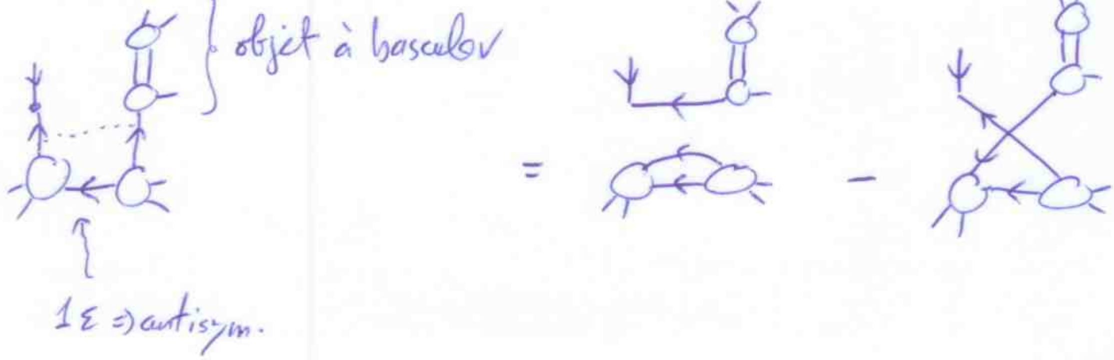


$\Rightarrow \quad C_2 = - C_2 - C_2 \quad \Rightarrow \quad \boxed{C_2 = 0}$

$\Rightarrow \quad \boxed{C = \frac{1}{2} FI}$

calcul de D par bascule encore:

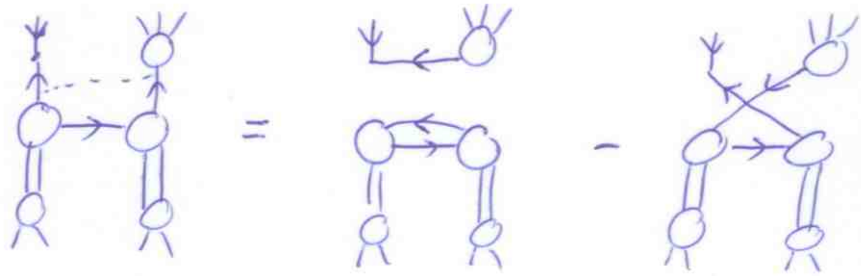
$D =$



soit $D = H^2 - D \quad \Rightarrow \quad \boxed{D = \frac{1}{2} H^2}$

Donc $\boxed{B = \frac{1}{2} F^2 I - \frac{1}{2} H^2}$

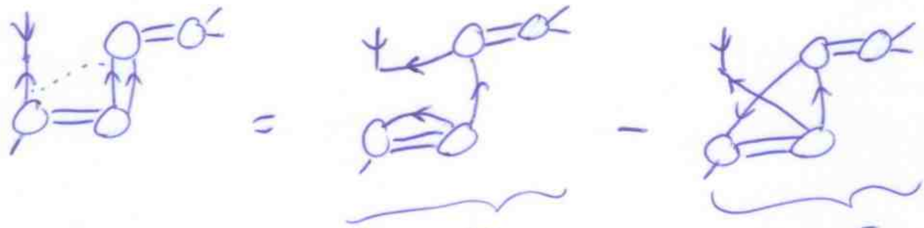
Maintenant on s'occupe de A :

$A =$ 

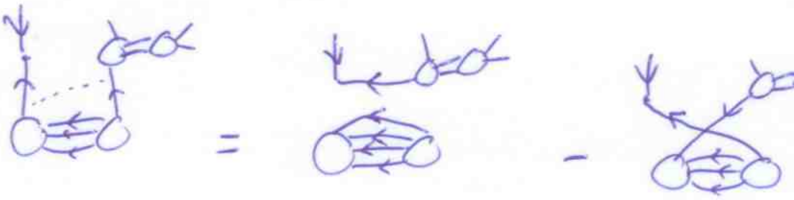
(variante de la bascule)

$\Rightarrow A = -F$  $- A \Rightarrow A = \frac{-1}{2} FE$

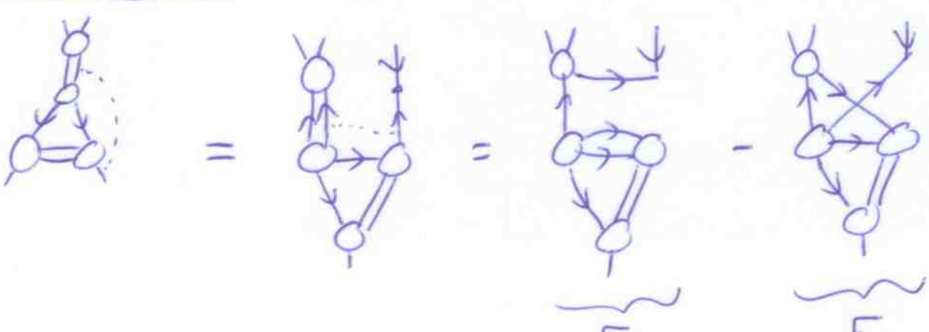
E

$E =$ 

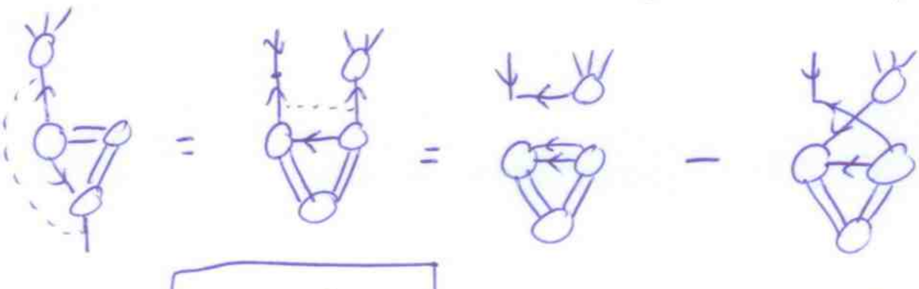
E_1 $-E_2$

$E_1 =$  $= HI - E_1$

$\Rightarrow \boxed{E_1 = \frac{1}{2} HI}$ par bascule encore.

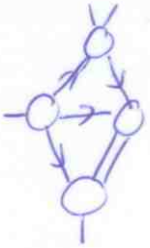
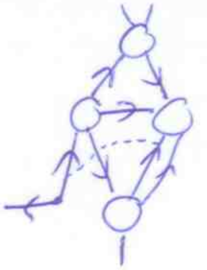
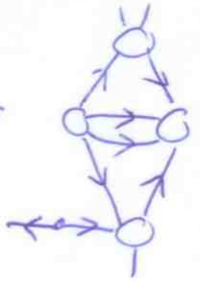
$E_2 =$ 

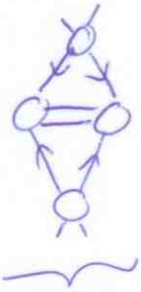
E_3 E_4

$E_3 =$  $= FJ - E_3$

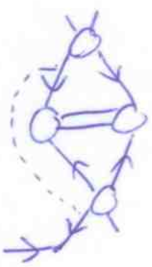
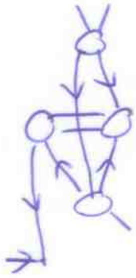
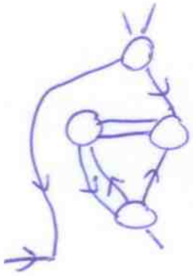
$\Rightarrow \boxed{E_3 = \frac{1}{2} FJ}$ par variante de bascule.

⑤

$E_4 =$  $=$  $=$  $-$ 

$=$  $- E_4 \quad \Rightarrow \quad E_4 = \frac{1}{2} E_5$

E_5

$E_5 =$  $=$  $-$  $= -E_4 + E_3$

$\Rightarrow \quad E_5 = -\frac{1}{2} E_5 + E_3 \quad \Rightarrow \quad E_5 = \frac{2}{3} E_3 = \frac{1}{3} FJ$

$\Rightarrow \quad E_4 = \frac{1}{6} FJ \quad \Rightarrow \quad E_2 = \frac{1}{2} FJ - \frac{1}{6} FJ = \frac{1}{3} FJ$

$\Rightarrow \quad E = \frac{1}{2} HI + \frac{1}{3} FJ$

et $A = -\frac{1}{2} F \left(\frac{1}{2} HI + \frac{1}{3} FJ \right) = -\frac{1}{4} FHI - \frac{1}{6} F^2 J$

et $T^2 = F \left(-\frac{1}{4} FHI - \frac{1}{6} F^2 J \right) + H \left(\frac{1}{2} F^2 I - \frac{1}{2} H^2 \right)$

càd $\boxed{T^2 = -\frac{1}{2} H^3 + \frac{1}{4} F^2 HI - \frac{1}{6} F^3 J}$

qui est la syzygie d'Hermite

⑥

En effet on fait le lien avec les notations de l'article de Weil et celui d'Hermite cité par Weil :

$$F = f = a x_1^4 + 4b x_1^3 x_2 + 6c x_1^2 x_2^2 + 4b' x_1 x_2^3 + a' x_2^4$$

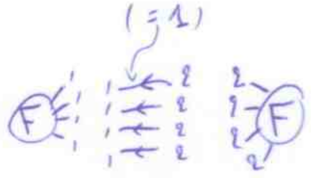
$$i = aa' - 4bb' + 3c^2$$

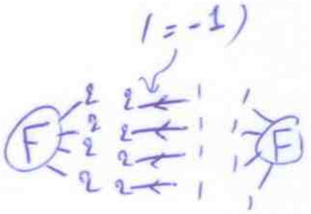
$$j = aca' + 2bcb' - ab'^2 - a'b^2 - c^3$$

$$g = (b^2 - ac) x_1^4 + \dots$$

$$h = (a^2 b' - 3abc + 2b^3) x_1^6 + \dots$$

formules tirées du papier d'Hermite

$I =$  $=$ 

$+$ 

$+$ termes mixtes, 1 & 2 présents ds chaque blob F.

$\overrightarrow{\text{développement}}$ des sommes sur les indices

$$= a\,(1)^4 a' + a'\,(-1)^4 a + \dots$$

$I = 2aa' + \dots$ ⇒ coeff de aa' ds I est 2

⇒ $\boxed{I = 2i}$

Coeff de x_1^4 ds H est :  $= 2F_{1111}F_{1122} - 2F_{1112}^2$

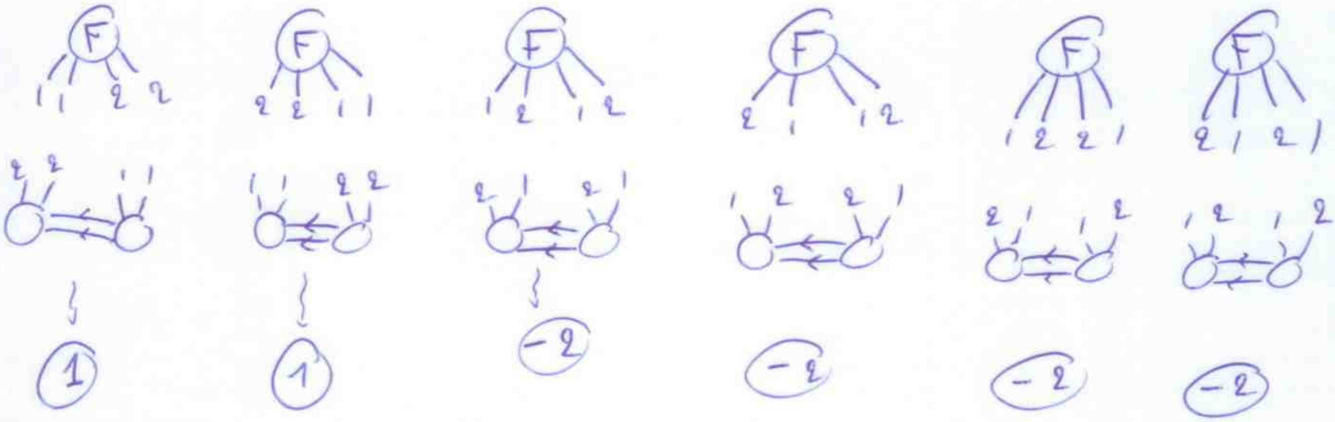
notation tensorielle

$= 2ac - 2b^2$

⇒ $\boxed{H = -2g}$

On cherche le coeff de c^3 dans J : $c = F_{1122} =$ 

6 possibilités pour blob du haut



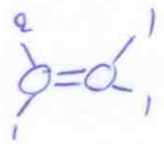
1 1 −2 −2 −2 −2

bas associé

façons d'obtenir c^3 avec signes

$$J = (\overbrace{1+1-2-2-2-2}^{-6})\,c^3 + \dots$$

$$\Rightarrow \quad J = -6c^3 + \dots \quad \Rightarrow \quad \boxed{J = 6j}$$

Coeff de x_1^6 dans T =  $= F_{1111}$  $- F_{1112}$ 

somme indice à développer

$$\underbrace{}_{b(2ac-2b^2)}$$

$= 2b^3$ + autre monomes.

$$\Rightarrow \quad \boxed{T = h}$$

syzygie $\Rightarrow$ $h^2 = -\frac{1}{2}(-2g)^3 + \frac{1}{4} f^2(-2g)(2i) - \frac{1}{6} f^3(6j)$

$$\Rightarrow \quad \boxed{h^2 = 4g^3 - if^2g - jf^3}$$ en notations Hermite-Weil.