Math 8450, Fall 2014, Topics in Mathematical Physics

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Location and times: TuTh from 12:30 pm to 1:45 pm, in KER 326.

Office hours: W from 10 am to 12 pm and by appointment.

Prerequisites: MATH 7310 or instructor permission. Recommended but not required: MATH 7360, MATH 7450.

Description: This course will be a self-contained introduction to rigorous statistical mechanics, with a view towards the mathematical theory of functional integrals from quantum field theory. Statistical mechanics can be viewed as the study of infinite systems of dependent random variables. An important features of such systems is the notion of phase transition which one can observe in the real world, such as the liquid-vapor transition of water or the ferromagnetic-paramagnetic transition in iron. Equilibrium statistical mechanics studies mathematicals models where such phenomena can be defined and analyzed rigorously. As examples of such models we will mostly consider spin systems, on a periodic lattice or on the continuum. The spins, or random variables, will be discrete (as in the Ising model) or continuous (as in lattice phi-four models). After a quick overview and the introduction of preliminaries from probability theory, we will develop the main mathematical techniques for the rigorous study of these systems, namely the method of cluster expansions and that of correlation inequalities. If time permits we will explain, on some examples, the connection to quantum mechanics, namely how some questions in quantum mechanics (spectral theory) can be rephrased as questions in statistical mechanics (probability theory) and vice versa. MATH 7360, Probability I, is desired but not required (if one knows basics of probability). The course will be independent of MATH 7450 taught during Spring 2014. However, for instance when discussing the quantum mechanics-statistical mechanics connection, one will need the contents of Chapter I of the textbook for MATH 7450 ("Quantum Mechanics and Quantum Field Theory, a Mathematical Primer" by J. Dimock), namely the statement without proof of the spectral theorem for self-adjoint operators. Also if time permits we will give an introduction to perturbative renormalization.

Course web page: http://people.virginia.edu/~aa4cr/Math8450F14.html

Assessment: Attendence and class participation are required and expected (30% or the course grade). Students will be asked to write lecture notes (in LaTeX) for this course, as a collaborative effort (40% or the course grade). Finally, students will also be asked to read a research article in the area and give an oral presentation at the end of the semester (30% or the course grade). There will be no final examination.

Textbook: We will not follow any particular textbook. However, please see the course webpage above for useful references and online resources.